The coupling of fermions to the three-dimensional noncommutative CP^{N-1} model: minimal and supersymmetric extensions

E. A. Asano,[1] H. O. Girotti,[2] M. Gomes,[1] A. Yu. Petrov,[1,*] A. G. Rodrigues,[1] and A. J. da Silva[1]

[1]*Instituto de Fisica, Universidade de São Paulo, Caixa Postal 66318, 05315-970, São Paulo - SP, Brazil*[†]

[2]*Instituto de Fisica, Universidade Federal do Rio Grande do Sul, Caixa Postal 15051, 91501-970 - Porto Alegre, RS, Brazil*[‡]

Abstract

We consider the coupling of fermions to the three-dimensional noncommutative CP^{N-1} model. In the case of minimal coupling, although the infrared behavior of the gauge sector is improved, there are dangerous (quadratic) infrared divergences in the corrections to the two point vertex function of the scalar field. However, using superfield techniques we prove that the supersymmetric version of this model with "antisymmetrized" coupling of the Lagrange multiplier field is renormalizable up to the first order in $\frac{1}{N}$. The auxiliary spinor gauge field acquires a nontrivial (nonlocal) dynamics with a generation of Maxwell and Chern-Simons noncommutative terms in the effective action. Up to the $1/N$ order all divergences are only logarithimic so that the model is free from nonintegrable infrared singularities.

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[*]Also at Department of Theoretical Physics, Tomsk State Pedagogical University Tomsk 634041, Russia (email: petrov@tspu.edu.ru)

[†]Electronic address: asano, mgomes, petrov, alexgr, ajsilva@fma.if.usp.br

[‡]Electronic address: hgirotti@if.ufrgs.br

I. INTRODUCTION

The renormalization problem is a central issue for the perturbative consistency of noncommutative (NC) field theories. This is of course true for any field theory but for the noncommutative setting renormalization becomes more stringent due to an unusual mixture of scales. In fact, a characteristic phenomena in such theories is the well known ultraviolet/infrared (UV/IR) mixing which, being the source of nonintegrable IR divergences [1] (for a review see [2]), destroys most of the perturbative schemes. It is therefore very important to find renormalizable noncommutative field theories free from the mentioned infrared divergences. We have recently proved that, at least up to next to leading order of $1/N$, this requirement is satisfied by the $(2+1)$ dimensional noncommutative version of the CP^{N-1} model if the basic field transforms in accord with the fundamental representation of the gauge group [3]. For the same model, we also investigated the situation where the basic field belongs to the adjoint representation. In contrast with the fundamental representation, we found that for the adjoint representation infrared divergences associated to nonplanar graphs are present. These infrared divergences indicate the breakdown of the model at higher orders of $1/N$. Our previous experience with the noncommutative versions of the four dimensional Wess-Zumino model [4] as well as with the (2+1) dimensional supersymmetric nonlinear sigma model [5], suggests that the overall behavior of the theory may be improved if fermions are included. In this paper we will investigate such possibility by coupling fermions to the gauge field either minimally or in a supersymmetric fashion. Of course, even in the case of minimal coupling, the fermionic field and its bosonic counterpart must belong to the same representation.

Very interesting results emerge from our analysis. As we shall prove, due to the induction of a Chern-Simons term, the gauge potential becomes much less singular. However, in the case of minimal coupling, in spite of the general smootheness of the gauge potential, the radiative corrections to the self energy of the scalar field are still plagued by nonintegrable infrared singularities. To evade this problem we then consider a supersymmetric extension of the model. This is done through the use of powerful superfield techniques [6, 7], which enable us to demonstrate the absence of the dangerous UV/IR mixing up to order $1/N$.

Our work is organized as follows. In Sec. II the inclusion of fermion fields minimally coupled to the gauge field is examined. In Sec. III the superfield formulation is introduced, we fix the notation to be employed and determine the propagators for the relevant fields. In Sec. IV we prove that the self-energy corrections of the scalar superfield are free from dangerous UV/IR mixing and in Sec. V give a general argument for the absence of these singularities in all Green functions up to $1/N$ order. A general overview of our results and the conclusions are contained in Sec. VI.

II. MINIMAL COUPLING OF FERMIONS TO THE CP^{N-1} MODEL

Assuming that the fermions have the same mass as their bosonic counterpart, the action associated to the model reads (for discussions on the commutative CP^{N-1} model see [8, 9, 10, 11, 12])

$$\int d^3x\mathcal{L} = \int d^3x[-(D_\mu\varphi)^\dagger * D^\mu\varphi - m^2\varphi^\dagger * \varphi - \bar{\psi} * \gamma_\mu D^\mu\psi - m\bar{\psi} * \psi + \mathcal{L}_\lambda], \qquad (2.1)$$

where φ_a and ψ_a, $a = 1, \ldots, N$ are scalar and two-component Dirac fields, respectively. They transform according to either the left fundamental or the adjoint representation of the gauge group. To keep uniformity throughout this work, we shall use the metric $g_{11} = g_{22} = -g_{00} = 1$ and the Dirac matrices to be employed in this section are $\gamma^0 = i\sigma^3$, $\gamma^1 = \sigma^1$ and $\gamma^2 = \sigma^2$, where the σ's are the Pauli matrices). The covariant derivative of the basic fields is $D_\mu\chi = \partial_\mu\chi + iA_\mu * \chi$, for $\chi = \varphi, \psi$ in the left fundamental representation, whereas $D_\mu\chi = \partial_\mu\chi + iA_\mu * \chi - i\chi * A_\mu$, for $\chi = \varphi, \psi$ in the adjoint representation. $\mathcal{L}_\lambda$ is the interaction Lagrangian which enforces a basic constraint for the φ_a fields; its possible forms will be given shortly. Besides, to evade unitarity problems, throughout this work we consider only space-space noncommutativity.

A. The bosonic model

We begin by recalling some basic results of the pure CP^{N-1} model, i.e. without fermions [3]:

(1) For the left fundamental representation case, with $\mathcal{L}_\lambda = \lambda * (\varphi * \varphi^\dagger - \frac{N}{g})$, the two point vertex functions of the gauge and λ fields are respectively:

$$F_b^{\mu\nu}(p) = -\frac{iN}{8\pi}(g^{\mu\nu}p^2 - p^\mu p^\nu)\int_0^1 dx\frac{(1-2x)^2}{M(x)}, \qquad (2.2)$$

and

$$F(p) = N\int\frac{d^3k}{(2\pi)^3}\frac{1}{(k+p)^2+m^2}\frac{1}{k^2+m^2} = \frac{iN}{8\pi}\int_0^1 dx\frac{1}{M(x)}, \qquad (2.3)$$

where $M(x) = [m^2 + p^2x(1-x)]^{1/2}$. Furthermore, the mixed two point vertex function F_μ of the A_μ and λ fields vanishes.

(2) For the adjoint representation there are two cases that have to be distinguished:

(2a) The part of the interaction Lagrangian which contains λ is $\mathcal{L}_\lambda = \lambda * [\varphi, \varphi^\dagger]_*$. Here also the mixed two point vertex function F_μ vanishes.

The two point vertex function of the A_μ field is

$$F_b^{\mu\nu}(p) = -\frac{iN}{4\pi}\Bigg\{(g^{\mu\nu}p^2 - p^\mu p^\nu)\int_0^1 dx\frac{(1-2x)^2}{M}(1-\mathrm{e}^{-M\sqrt{\tilde{p}^2}}) + 4\frac{\tilde{p}^\mu\tilde{p}^\nu}{\tilde{p}^2}\int_0^1 dx(\frac{1}{\sqrt{\tilde{p}^2}} + M)\mathrm{e}^{-M\sqrt{\tilde{p}^2}}\Bigg\}, \qquad (2.4)$$

in which $\tilde{p}_\mu = \theta_{\mu\nu}p^\nu$ and $\theta_{\mu\nu}$ is the constant antisymmetric matrix characterizing the noncommutativity of the underlying space. Notice that the above result is transversal but possesses an infrared singularity at $\tilde{p} = 0$.

The two point vertex function of the λ field is modified to

$$2F(p) + F_{NP}(p) \equiv \frac{iN}{4\pi}f(p), \qquad (2.5)$$

where F was given in (2.3) and the nonplanar part F_{NP} is

$$F_{NP}(p) = -\frac{iN}{4\pi}\int_0^1 dx\,\frac{\mathrm{e}^{-M\sqrt{\tilde{p}^2}}}{M}. \tag{2.6}$$

The function $f(p)$ is explicitly given by

$$f(p) = \int_0^1 dx \frac{1-\mathrm{e}^{-M\sqrt{\tilde{p}^2}}}{M} \approx \begin{cases} \sqrt{\tilde{p}^2} & \text{for } p \to 0, \\ \pi/\sqrt{p^2} & \text{for } p^2 \gg m^2. \end{cases} \tag{2.7}$$

For future use it is convenient to identify

$$f(p) = -16\pi i \int \frac{d^3k}{(2\pi)^3}\, I(k,p), \tag{2.8}$$

where

$$I(k,p) = \frac{\sin^2(k\wedge p)}{(k^2+m^2)((k+p)^2+m^2)} \tag{2.9}$$

and $k\wedge p = \frac{1}{2}k\cdot\tilde{p}$.

(2b) The interaction Lagrangian $\mathcal{L}_\lambda$ has the same form as in the case of the left fundamental representation. The two point vertex functions of the A_ν and λ fields are still given by (2.4) and (2.3) but now there exists a nonvanishing mixed two point vertex function

$$\begin{aligned} F_\mu(p) &= N\int \frac{d^3k}{(2\pi)^3}\,\frac{(2k+p)_\mu}{(k^2+m^2)[(k+p)^2+m^2]}\mathrm{e}^{-i2k\wedge p} \\ &= \frac{N\tilde{p_\mu}}{4\pi\sqrt{\tilde{p}^2}}\int_0^1 dx \mathrm{e}^{-M\sqrt{\tilde{p}^2}} \equiv \frac{Ng(p)}{4\pi}\tilde{p}_\mu. \end{aligned} \tag{2.10}$$

B. Including fermions

Due to the inclusion of fermionic fields, the two point vertex function of the A_μ field receives a new contribution:

$$F_f^{\mu\nu}(p) = -N\int \frac{d^3k}{(2\pi)^3}\mathrm{Tr}[\gamma^\nu \frac{i}{-i\not{k}+m}\gamma^\mu \frac{i}{-i(\not{k}+\not{p})+m}]J(k,p), \tag{2.11}$$

where $J(k,p)$ is either equal to one or to $4\sin^2(k\wedge p)$ for the left fundamental or the adjoint representations, respectively. In (2.11) the subscript f was used to designate the fermionic part. After some standard manipulations, we arrive at

$$\begin{aligned} F_f^{\mu\nu}(p) &= -2N\int_0^1 dx \int \frac{d^3k}{(2\pi)^3}J(k,p) \\ &\times \frac{2k^\mu k^\nu - 2p^\mu p^\nu x(1-x) - g^{\mu\nu}[k^2 - p^2x(1-x)+m^2] + im\epsilon^{\mu\nu\rho}p_\rho}{[k^2+p^2x(1-x)+m^2]^2}. \end{aligned} \tag{2.12}$$

For the left fundamental representation this produces the well known result [13, 14]

$$F_f^{\mu\nu}(p) = -\frac{Ni}{2\pi}(g^{\mu\nu}p^2 - p^\mu p^\nu)\int_0^1 dx\frac{x(1-x)}{M} + \frac{mN}{4\pi}\epsilon^{\mu\nu\rho}p_\rho\int_0^1 dx\frac{1}{M}. \tag{2.13}$$

For the adjoint representation, the use of $\sin^2(k\wedge p) = \frac{1-\cos(2k\wedge p)}{2}$ leads to a planar contribution equal to twice the above result. The nonplanar contribution (which contains the factor $\cos(2k\wedge p)$) gives

$$\begin{aligned} F_{NPf}^{\mu\nu}(p) &= \frac{iN}{\pi}(g^{\mu\nu}p^2 - p^\mu p^\nu)\int_0^1 dx\frac{x(1-x)}{M}\mathrm{e}^{-M\sqrt{\tilde{p}^2}} + \frac{iN}{\pi}\frac{\tilde{p}^\mu\tilde{p}^\nu}{\tilde{p}^2}\int_0^1 dx(M+\frac{1}{\sqrt{\tilde{p}^2}})\mathrm{e}^{-M\sqrt{\tilde{p}^2}} \\ &\quad -\frac{mN}{2\pi}\epsilon^{\mu\nu\rho}p_\rho\int_0^1 dx\frac{1}{M}e^{-M\sqrt{\tilde{p}^2}}. \end{aligned} \tag{2.14}$$

Thus, by adding the contributions from the bosonic and fermionic loops we get the total two point vertex function of the gauge field as being:

1. For the left fundamental representation (sum of Eqs. (2.2) plus (2.13))

$$F^{\mu\nu}(p) = \frac{-iN}{8\pi}[(g^{\mu\nu}p^2 - p^\mu p^\nu) + 2im\epsilon^{\mu\nu\rho}p_\rho]\int_0^1\frac{dx}{M}. \tag{2.15}$$

2. For the adjoint representation (sum of Eqs. (2.4) plus twice (2.13) plus (2.14))

$$F^{\mu\nu}(p) = \frac{-iN}{4\pi}f(p)[(g^{\mu\nu}p^2 - p^\mu p^\nu) + 2im\epsilon^{\mu\nu\rho}p_\rho]. \tag{2.16}$$

As can be seen, $F^{\mu\nu}(p)$ behaves smoothly as p tends to zero. Notice the presence of terms proportional to $\epsilon^{\mu\nu\rho}$ in Eqs. (2.13) and (2.16), which in the effective action correspond to the bilinear part of the noncommutative Chern-Simons term. From now on we will restrict our considerations to the adjoint representation.

For the case (2a) the propagator for the λ field is $\Delta(p) = \frac{4\pi i}{Nf(p)}$ and the propagator for the gauge field in the Landau gauge is

$$\Delta_{\mu\nu}(p) = \frac{-4\pi i}{Nf(p)(p^2+4m^2)}[(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) - \frac{2im}{p^2}\epsilon_{\mu\nu\rho}p^\rho]. \tag{2.17}$$

For the case (2b), due to the nonvanishing mixed two point vertex function of the λ and A_μ fields, the computation of the gauge field propagator is much more involved than in the previous case. We find (also in the Landau gauge)

$$\begin{aligned} \Delta_{\mu\nu}(p) &= A_1(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) + A_2\tilde{p}_\mu\tilde{p}_\nu + A_3\bar{p}_\mu\bar{p}_\nu + A_4(\tilde{p}_\mu\bar{p}_\nu - \tilde{p}_\nu\bar{p}_\mu) + A_5\epsilon_{\mu\nu\rho}p^\rho \\ &= (A_1 - p^2\tilde{p}^2A_3)(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) + (A_2 + p^2A_3)\tilde{p}_\mu\tilde{p}_\nu + (A_5 + \tilde{p}^2A_4)\epsilon_{\mu\nu\alpha}p^\alpha, \end{aligned} \tag{2.18}$$

where $\bar{p}_\alpha \equiv \epsilon_{\alpha\beta\gamma}p^\beta\tilde{p}^\gamma$ and the coefficients A_i, $i = 1, \ldots, 5$, are functions of p:

$$A_1 = -i\frac{4\pi}{N}\frac{1}{f(p)}\frac{1}{p^2+4m^2}, \qquad A_2 = \frac{4\pi}{N}\frac{g^2(p)}{h(p)}\frac{1}{p^2+4m^2}, \tag{2.19}$$

$$A_3 = \frac{4\pi}{N}\frac{4m^2 g^2(p)}{h(p)(p^2)^2}\frac{1}{p^2+4m^2}, \qquad A_4 = i\frac{4\pi}{N}\frac{2mg^2(p)}{h(p)p^2}\frac{1}{p^2+4m^2}, \tag{2.20}$$

and

$$A_5 = -\frac{4\pi}{N}\frac{2m}{f(p)}\frac{1}{p^2(p^2+4m^2)}, \tag{2.21}$$

where

$$h(p) = -if(p)\left[\tilde{p}^2 g^2(p) + f^2(p)(p^2+4m^2)\right]. \tag{2.22}$$

For large momenta this propagator coincides with that in Eq. (2.17), since $g(p)$ exponentially decreases or strongly oscillates in that limit.

Notice that in both situations the gauge propagator is much less singular than in the pure CP^{N-1} model. This smoothness of the infrared behavior comes as a direct effect of the generation of the Chern-Simons term which provided the displacement from the origin of the usual ($p^2 = 0$) singularity.

For reference we also quote the expressions for the λ and mixed (λ, A_ν) propagators

$$\Delta(p) = \frac{4\pi}{N}\frac{f^2(p)}{h(p)}(p^2+4m^2), \qquad \Delta_\nu(p) = -\frac{4\pi}{N}\frac{f(p)g(p)}{h(p)}(i\tilde{p}_\nu + \frac{2m\bar{p}_\nu}{p^2}). \tag{2.23}$$

Although we have improved the infrared behavior of the A_μ propagator we still get trouble with the radiative corrections to the propagator for the φ field. In fact, whereas graph 1b is finite (in the Landau gauge), a direct calculation shows that the nonplanar parts of the graphs of Figs. 1a and 1c are infrared quadratically divergent. Up to the $1/N$ order they are the only infrared divergent diagrams contributing to the self energy of φ field. The sum of their nonplanar parts does not vanish and therefore, at higher orders, leads to a breakdown of the $1/N$ expansion [3]. To overcome this problem a further extension of the model is needed. This will be the subject of the following sections where we discuss a supersymmetric extension of the noncommutative CP^{N-1} model.

III. THE NONCOMMUTATIVE SUPERSYMMETRIC CP^{N-1} MODEL

In the adjoint representation the noncommutative superfield generalization of the CP^{N-1} model is described by (see also [15, 16] for supersymmetric extensions of its commutative counterpart)

$$\begin{aligned} S = & -\int d^5z\Big[\frac{1}{2}(D^\alpha\bar{\phi}_a + i[\bar{\phi}_a, A^\alpha]_*) * (D_\alpha\phi_a - i[A_\alpha,\phi_a]_*) + m\bar{\phi}_a\phi_a \\ & + \eta * (a[\bar{\phi}_a,\phi_a]_* + b\{\bar{\phi}_a,\phi_a\}_*) - \eta\frac{Nb}{g}\Big], \end{aligned} \tag{3.1}$$

where ϕ_a with $a = 1 \ldots N$ is a set of scalar (super) fields, $\bar{\phi}_a$ are their complex conjugated ones, A_α is a two-component spinor gauge field and η is a Lagrange multiplier superfield

which implements the constraint $\{\bar{\phi}_a, \phi_a\}_* \equiv \bar{\phi}_a * \phi_a + \phi_a * \bar{\phi}_a = \frac{N}{g}$; a and b are parameters which control the two possible orderings of the trilinear term containing the η, ϕ and $\bar{\phi}$ fields. Hereafter, we employ the same notation and definitions of [7] (see also a description of the three-dimensional superfield approach in [17]). Concisely, $l^2 \equiv \frac{1}{2} l^\alpha l_\alpha = \frac{1}{2} C^{\alpha\beta} l_\beta l_\alpha$ for any spinor l^α (and $D^2 = \frac{1}{2} D^\alpha D_\alpha$), with $C_{\alpha\beta} = -C^{\alpha\beta}$ an antisymmetric matrix normalized as $C_{12} = -i$, $\psi_\alpha = \psi^\beta C_{\beta\alpha}$ and $\psi^\alpha = C^{\alpha\beta}\psi_\beta$. The Dirac matrices with both spinor indices upstairs are $\gamma^m = (\mathbf{1}, \sigma^3, -\sigma^1)$ and satisfy $\{\gamma^m, \gamma^n\} = 2g^{mn}\mathbf{1}$.

The above action is invariant under the infinitesimal supergauge transformation:

$$\delta\phi = i[K,\, \phi]_*\,, \qquad \delta\eta = i[K,\, \eta]_*, \qquad \delta A_\alpha = D_\alpha K + i[K, A_\alpha]_*, \tag{3.2}$$

where K is the scalar superfield gauge parameter. We will consider two cases, namely, the *commutator* case when $a = 1$ and $b = 0$ and the *anticommutator* case when $a = 0$ and $b = 1$. Notice that dynamical generation of mass only occurs in the anticommutator case (the analysis is entirely similar to the one in [5]). In the sequel we will be explicitly analyzing the commutator case but we will also comment on the other possibility.

As it is well known, charge conjugation (and parity) are in general broken for noncommutative field theories [18]. Notice however, that for the commutator case the above action is invariant under the "charge conjugation" transformation $\phi \leftrightarrow \bar{\phi}$, $A_\alpha \to A_\alpha$, and $\eta \to -\eta$ and, as a consequence, the "mixed propagator" $< \eta A_\alpha >$ vanishes. This last conclusion depends crucially on the way in which the η and ϕ fields are coupled. Had we used an anticommutator in the term multiplying the η field, then η would be even under charge conjugation and the mixed propagator would not vanish. For the commutator case, an equivalent and useful form for the above action is

$$\begin{aligned} S \;=\; & \int d^5z \Big[\bar{\phi}_a(D^2 - m)\phi_a - \frac{i}{2}([\bar{\phi}_a, A^\alpha]_* * D_\alpha\phi_a - D^\alpha\bar{\phi}_a * [A_\alpha, \phi_a]_*) \\ & -\frac{1}{2}[\bar{\phi}_a, A^\alpha]_* * [A_\alpha, \phi_a]_* - \eta * [\bar{\phi}_a, \phi_a]_*\Big]. \end{aligned} \tag{3.3}$$

As in the pure CP^{N-1} model, at the classical level only the scalar fields are dynamical but quantum corrections may provide effective dynamics for the other fields (compare also with [5]). All fields belong to the adjoint representation of the gauge group which explains the commutators in the terms involving the gauge field; these commutators cause sine factors in the corresponding vertices of the Feynman graphs. Using that $(D^2)^2 = \Box$ and $(D^2+m)(-D^2+m) = -\Box + m^2$ we obtain the free propagator for the scalar fields

$$< T\bar{\phi}_a(z_1)\phi_b(z_2) >= i\delta_{ab}\frac{D^2 + m}{\Box - m^2}\delta^5(z_1 - z_2), \tag{3.4}$$

which, in momentum space reads

$$< T\bar{\phi}_a(k_1, \theta_1)\phi_b(k_2, \theta_2) >= (2\pi)^3\delta^3(k_1 + k_2) < \bar{\phi}_a(k_1, \theta_1)\phi_b(-k_1, \theta_2) >, \tag{3.5}$$

where

$$< \bar{\phi}_a(k, \theta_1)\phi_b(-k, \theta_2) >= -i\delta_{ab}\frac{D^2 + m}{k^2 + m^2}\delta_{12}, \tag{3.6}$$

with $\delta_{12} \equiv \delta^2(\theta_1 - \theta_2)$.

Let us now obtain the effective propagators for the η and A^α fields. First we turn to the η field. The effective propagator is generated by the supergraph of Fig. 2. The contribution of this graph to the effective action $S_2 = \int \frac{d^3p}{(2\pi)^3} S_2(p)$ is

$$
\begin{aligned}
iS_2(p) &= 2N \int d^2\theta_1 d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} I(k,p) \\
&\times (D^2+m)\delta_{12}(D^2+m)\delta_{12}\eta(-p,\theta_1)\eta(p,\theta_2).
\end{aligned} \tag{3.7}
$$

Performing D-algebra transformations in a way analogous to the derivation of the effective propagator for Σ field in [5], we get

$$
S_2(p) = \frac{N}{8\pi} f(p) \int d^2\theta \, \eta(-p,\theta)(D^2+2m)\eta(p,\theta), \tag{3.8}
$$

where $f(p)$ was defined in Eq. (2.7). From this expression we can obtain the propagator for the η field:

$$
< \eta(p,\theta_1)\eta(-p,\theta_2) >= -\frac{4\pi i}{N} \frac{D^2 - 2m}{f(p)(p^2+4m^2)} \delta_{12}. \tag{3.9}
$$

This propagator is linearly divergent for small p, since in this limit $f(p) \approx \sqrt{\tilde{p}^2}$. However, this divergence does not bring difficulties since, for zero momentum, the radiative corrections to the two point vertex function of the η field will also vanish (as a consequence of the sine factors at the vertices). On the other hand, for high momenta the nonplanar contribution in Eq. (3.9) rapidly decreases. Therefore, when analyzing the ultraviolet behavior of Feynman amplitudes we can take just the asymptotic behavior of the planar part which furnishes

$$
< \eta(p,\theta_1)\eta(-p,\theta_2) >\approx -\frac{4i}{N} \frac{D^2 - 2m}{\sqrt{p^2}} \delta_{12}. \tag{3.10}
$$

Next, we turn to the effective propagator of the spinor field A_α. It is formed by the two contributions shown in Fig. 3. The first graph, depicted in Fig 3a, gives the following contribution:

$$
\begin{aligned}
iS_{3a}(p) &= -\int d^2\theta_1 d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} A^\alpha(-p,\theta_1) A^\beta(p,\theta_2) \sin^2(k \wedge p) \\
&\times \Big[D_{\alpha 1} < \phi_a(-k,\theta_1)\bar{\phi}_b(k,\theta_2) > (< \bar{\phi}_a(k+p,\theta_1)\phi_b(-k-p,\theta_2) > \overleftarrow{D}_{\beta 2}) \\
&- (D_{\alpha 1} < \phi_a(-k,\theta_1)\bar{\phi}_b(k,\theta_2) > \overleftarrow{D}_{\beta 2}) < \bar{\phi}_a(k+p,\theta_1)\phi_b(-k-p,\theta_2) > \Big],
\end{aligned} \tag{3.11}
$$

where the notation $D_{\gamma i}$ was used to indicate that the supercovariant derivative is applied to the field whose Grassmanian argument is θ_i. Taking into account the explicit form of the propagators, we have

$$
\begin{aligned}
iS_{3a}(p) \;&=\; N\int d^2\theta_1 d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} A^\alpha(-p,\theta_1)A^\beta(p,\theta_2)\sin^2(k\wedge p)\\
&\times\; \Big[\frac{D_{\alpha 1}(D_1^2+m)}{k^2+m^2}\delta_{12}\frac{(D_1^2+m)D_{\beta 2}}{(k+p)^2+m^2}\delta_{12}\\
&-\; \frac{D_{\alpha 1}(D_1^2+m)D_{\beta 2}}{k^2+m^2}\delta_{12}\frac{D_1^2+m}{(k+p)^2+m^2}\delta_{12}\Big].
\end{aligned}
\tag{3.12}
$$

Integrating by parts some of the spinor derivatives and by using the identity $D_{\beta 2}(k,\theta_2)\delta_{12} = -D_{\beta 1}(-k,\theta_1)\delta_{12}$ we arrive at

$$
\begin{aligned}
iS_{3a}(p) \;&=\; N\int d^2\theta_1 d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} I(k,p)\\
&\times\; \Big[2(D_1^2+m)\delta_{12}D_{\alpha 1}(D_1^2+m)D_{\beta 1}\delta_{12}A^\alpha(-p,\theta_1)A^\beta(p,\theta_2)\\
&+\; (D_1^2+m)\delta_{12}(D_1^2+m)D_{\beta 1}\delta_{12}(D^\alpha A_\alpha)(-p,\theta_1)A^\beta(p,\theta_2)\Big].
\end{aligned}
\tag{3.13}
$$

It is convenient to separate S_{3a} into two parts, $S_{3a} = S_{3a}^{(1)} + S_{3a}^{(2)}$, where $iS_{3a}^{(1)}$ and $iS_{3a}^{(2)}$ are associated to the two terms in the large brackets of (3.13). Consider first $iS_{3a}^{(1)}$ which, after transporting D^2 from one of the propagators to the other factors, becomes

$$
\begin{aligned}
iS_{3a}^{(1)}(p) \;&=\; N\int d^2\theta_1 d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} I(k,p)\\
&\times\; \Big[2m\delta_{12}D_{\alpha 1}(D_1^2+m)D_{\beta 1}\delta_{12}A^\alpha(-p,\theta_1)A^\beta(p,\theta_2)\\
&+\; 2\delta_{12}D_1^2\Big(D_{\alpha 1}(D_1^2+m)D_{\beta 1}\delta_{12}A^\alpha(-p,\theta_1)\Big)A^\beta(p,\theta_2)\Big].
\end{aligned}
\tag{3.14}
$$

Now we employ the identity $\{D_{\alpha 1}, D_1^2\} = 0$ [7] which leads to

$$
\begin{aligned}
iS_{3a}^{(1)}(p) \;&=\; N\int d^2\theta_1 d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} I(k,p)\\
&\times\; \Big[2\delta_{12}(k^2+m^2)D_{\alpha 1}D_{\beta 1}\delta_{12}A^\alpha(-p,\theta_1)A^\beta(p,\theta_2)\\
&+\; 2\delta_{12}(-D_1^2+m)D_{\alpha 1}D_{\beta 1}\delta_{12}(D^2A^\alpha(-p,\theta_1))A^\beta(p,\theta_2)\Big].
\end{aligned}
\tag{3.15}
$$

The use of the relationship

$$
D_\alpha(-k,\theta_1)D_\beta(-k,\theta_1) = k_{\alpha\beta} - C_{\alpha\beta}D^2(-k,\theta_1)
\tag{3.16}
$$

now provides

$$
\begin{aligned}
iS_{3a}^{(1)}(p) \;&=\; 2N \int d^2\theta_1 d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} I(k,p) \\
&\times\; \Big[\delta_{12}(k^2+m^2)(k_{\alpha\beta} - C_{\alpha\beta}D^2)\delta_{12}A^\alpha(-p,\theta_1)A^\beta(p,\theta_2) \\
&+\; \delta_{12}(-D^2+m)(k_{\alpha\beta} - C_{\alpha\beta}D^2)\delta_{12}(D^2A^\alpha(-p,\theta_1))A^\beta(p,\theta_2)\Big]. \qquad (3.17)
\end{aligned}
$$

The only terms giving non-zero contributions are those containing just one D^2 since $\delta_{12}D^2\delta_{12} = \delta_{12}$. Indeed, by employing this identity and after integrating over θ_2 with the help of the delta function, we obtain

$$
\begin{aligned}
iS_{3a}^{(1)}(p) \;&=\; -2N \int d^2\theta \int \frac{d^3k}{(2\pi)^3} I(k,p) \qquad (3.18) \\
&\times\; \Big[(k^2+m^2)C_{\alpha\beta}A^\alpha(-p,\theta)A^\beta(p,\theta) + (k_{\alpha\beta} + mC_{\alpha\beta})(D^2A^\alpha(-p,\theta))A^\beta(p,\theta)\Big].
\end{aligned}
$$

The second term of (3.13) is

$$
\begin{aligned}
iS_{3a}^{(2)}(p) \;&=\; N \int d^2\theta_1 d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} I(k,p) \\
&\times\; \Big[(D_1^2+m)\delta_{12}(D_1^2+m)D_{\beta 1}\delta_{12}(D^\alpha A_\alpha)(-p,\theta_1)A^\beta(p,\theta_2)\Big]. \qquad (3.19)
\end{aligned}
$$

In this expression we must keep only the term proportional to $D_1^2\delta_{12}(D_1^2+m)D_{\beta 1}\delta_{12}$ (the remaining part is a trace of an odd number of derivatives and therefore vanishes). Thus, after manipulations similar to those done for $S_{3a}^{(1)}$, we find

$$
iS_{3a}^{(2)}(p) = -N \int d^2\theta \int \frac{d^3k}{(2\pi)^3} I(k,p) \Big[D^\gamma D^\alpha A_\alpha(-p,\theta)(k_{\gamma\beta} + mC_{\gamma\beta})A^\beta(p,\theta)\Big]. \qquad (3.20)
$$

By adding (3.18) and (3.20) we can write the total contribution from Fig. $3a$ as

$$
\begin{aligned}
iS_{3a}(p) \;&=\; -2N \int d^2\theta \int \frac{d^3k}{(2\pi)^3} I(k,p) \\
&\times\; \Big[(k^2+m^2)C_{\alpha\beta}A^\alpha(-p,\theta)A^\beta(p,\theta) + (k_{\alpha\beta} + mC_{\alpha\beta})(D^2A^\alpha(-p,\theta))A^\beta(p,\theta) \\
&+\; \frac{1}{2}D^\gamma D^\alpha A_\alpha(-p,\theta)(k_{\gamma\beta} + mC_{\gamma\beta})A^\beta(p,\theta)\Big]. \qquad (3.21)
\end{aligned}
$$

The algebraic manipulations for the graph $3b$ are considerably simpler and yield

$$
iS_{3b}(p) \;=\; 2N \int \frac{d^3k}{(2\pi)^3} \frac{\sin^2(k\wedge p)}{(k+p)^2+m^2} C_{\alpha\beta}A^\alpha(-p,\theta)A^\beta(p,\theta). \qquad (3.22)
$$

The complete two point vertex function for the A_α field is the sum of (3.21) and (3.22) and therefore reads

$$
\begin{aligned}
iS_3(p) &= -2N\int d^2\theta \int \frac{d^3k}{(2\pi)^3} I(k,p) \\
&\times\ (k_{\gamma\beta}+mC_{\gamma\beta})\Big[(D^2A^\gamma(-p,\theta))A^\beta(p,\theta)+\frac{1}{2}D^\gamma D^\alpha A_\alpha(-p,\theta)A^\beta(p,\theta)\Big]. \qquad (3.23)
\end{aligned}
$$

Observe that the dangerous linear divergences (as well as the logarithimic ones) present in S_{3a} and S_{3b} were canceled in the above result (compare with [19]). As a consequence the free two point vertex function of the gauge field does not present UV/IR mixing. Furthermore, notice that the graphs in the Figs. 2 and 3 cannot occur as subgraphs of more complicated diagrams, i.e. they are "illegal" subgraphs, since they have already been taken into account to construct the propagators for the A^α and η fields.

The two point vertex function (3.23) allows us to find the effective propagator. By recalling (2.8) and using that

$$
\int \frac{d^3k}{(2\pi)^3} I(k,p)k_{\alpha\beta} = -\frac{p_{\alpha\beta}}{2}\int \frac{d^3k}{(2\pi)^3} I(k,p), \qquad (3.24)
$$

we obtain

$$
S_3(p) = \frac{N}{16\pi}\int d^2\theta f(p)[-p_{\gamma\beta}+2m\ C_{\gamma\beta}]A^\beta(p,\theta)W^\gamma(-p,\theta), \qquad (3.25)
$$

where W^γ is the linear part of the field strength, i.e.,

$$
W^\gamma = \frac{1}{2}D^\alpha D^\gamma A_\alpha = \frac{1}{2}D^\gamma D^\alpha A_\alpha + D^2 A^\gamma. \qquad (3.26)
$$

After some straightforward manipulations Eq. (3.25) can be rewritten as

$$
\begin{aligned}
S_3(p) &= \frac{N}{16\pi}\int d^2\theta f(p)A^\beta(p,\theta)[D^2+2m]W_\beta(-p,\theta) \\
&= \frac{N}{16\pi}\int d^2\theta f(p)[W^\alpha W_\alpha + 2mW^\alpha A_\alpha], \qquad (3.27)
\end{aligned}
$$

which is, of course, invariant under the linearized gauge transformation $\delta A^\alpha = D^\alpha K$. The two terms in the last equality in Eq. (3.27) are nonlocal versions of the Maxwell and Chern-Simons actions. In the commutative situation the effective action also contains nonlocal Maxwell and Chern-Simons terms but in contrast with the above result in that case the leading small p terms are local.

For quantization the above gauge freedom must be eliminated. This is done by adding to (3.25) the following gauge fixing action

$$
S_{GF}(p) = \frac{N}{32\pi\xi}\int d^2\theta f(p)D^\beta A_\beta(p,\theta)D^2D^\alpha A_\alpha(-p,\theta). \qquad (3.28)
$$

Hence the pure gauge total quadratic action is

$$S_{A^\alpha}(p) = -\frac{N}{32\pi}\int \frac{d^3p}{(2\pi)^3} d^2\theta f(p) A_\alpha(-p,\theta)[D^\beta D^\alpha(D^2+2m) + \frac{1}{\xi} D^\alpha D^\beta D^2] A_\beta(p,\theta). \quad (3.29)$$

This leads to the following effective propagator

$$< A^\alpha(p,\theta_1) A^\beta(-p,\theta_2) > \; = \; \frac{4\pi i}{N f(p)} \left[\frac{(D^2-2m)D^\beta D^\alpha}{p^2(p^2+4m^2)} + \xi \frac{D^2 D^\alpha D^\beta}{(p^2)^2} \right] \delta_{12}, \quad (3.30)$$

which can also be written as

$$\begin{aligned} < A^\alpha(p,\theta_1) A^\beta(-p,\theta_2) > \; = \; & \frac{4\pi i}{N f(p)} \Big[- \frac{2mp^{\alpha\beta}}{p^2(p^2+4m^2)} + (\frac{1}{p^2+4m^2} - \frac{\xi}{p^2}) C^{\alpha\beta} \\ & + \frac{1}{p^2}(\frac{1}{p^2+4m^2} + \frac{\xi}{p^2}) p^{\alpha\beta} D^2 + \frac{2mC^{\alpha\beta}}{p^2(p^2+4m^2)} D^2 \Big] \delta_{12}. \quad (3.31) \end{aligned}$$

As for low momenta $f(p) \simeq \sqrt{\tilde{p}^2}$ then the effective propagator (3.31) behaves as $1/p^3$. Nevertheless, as in the nonsupersymmetric model, due to the sine factors in the vertices no infrared divergence should arise from such behavior.

Aiming to a detailed investigation of the renormalization properties of the model we now examine the UV limit of the above propagator. For high momenta we need to consider only the planar contributions as the nonplanar ones decay very rapidly. In this situation $f(p) \simeq \pi/\sqrt{p^2}$ so that

$$< A^\alpha(p,\theta_1) A^\beta(-p,\theta_2) > \simeq \frac{4i}{N} [\frac{1-\xi}{(p^2)^{1/2}} C^{\alpha\beta} + \frac{1+\xi}{(p^2)^{3/2}} p^{\alpha\beta} D^2] \delta_{12}. \quad (3.32)$$

For $\xi = -1$ this expression assumes the simpler form

$$< A^\alpha(p,\theta_1) A^\beta(-p,\theta_2) > = \frac{8i}{N} \frac{C^{\alpha\beta}}{(p^2)^{1/2}} \delta_{12}. \quad (3.33)$$

The action of the Faddeev-Popov ghosts is

$$S_{FP} = -\frac{N}{32\pi}\int d^3p d^2\theta f(p)(c' D^2 c - ic' D^\alpha [A_\alpha, c]_*), \quad (3.34)$$

yielding the ghost propagator

$$< c'(p,\theta_1) c(-p,\theta_2) > = -i \frac{32\pi}{N} \frac{D^2}{p^2 f(p)} \delta_{12}. \quad (3.35)$$

A direct consideration of the supergraphs involving ghost loops shows that they will begin to contribute only in the $\frac{1}{N^2}$ order.

In the anticommutator case we notice that the two point vertex functions of ϕ, A_α and the planar part of the η fields are the same as we calculated before but the nonplanar part of the

two point vertex function of the η field changes sign. Besides that, the additional effective action

$$S_{A\eta} = -\frac{N}{8\pi}\int_0^1 dx \frac{\mathrm{e}^{-M\sqrt{\tilde{p}^2}}}{\sqrt{\tilde{p}^2}}\tilde{p}_{\beta\gamma}A^\gamma(-p,\theta)D^\beta\eta(p,\theta), \tag{3.36}$$

coming from the graph in Fig. 4, is induced, leading to a nonvanishing mixed propagator $< A_\alpha \eta >$. From the above expression one sees that any graph containing the mixed propagator is superficially convergent; thus, to analyze the UV behavior of the Green functions we can discard such graphs and use the same propagators as in commutator case.

IV. RADIATIVE CORRECTIONS TO THE TWO POINT VERTEX FUNCTION OF THE SCALAR FIELD

At the next to leading order in $\frac{1}{N}$ the divergentt contributions to the two point vertex function of ϕ field are given by the graphs shown in Fig. 1, where continuos, wavy and dashed lines now represent the propagators for the ϕ, A_α and η superfields. Using the propagator in Eq. (3.9) for η field, the amplitude for the graph in Fig. 1a is

$$\begin{aligned} iS_{1a}(p) &= \frac{16\pi}{N}\int\frac{d^3k}{(2\pi)^3}\int d^2\theta_1 d^2\theta_2\phi_a(-p,\theta_1)\bar{\phi}_a(p,\theta_2)\frac{\sin^2(k\wedge p)}{((k+p)^2+m^2)f(k)(k^2+4m^2)} \\ &\times (D^2-2m)\delta_{12}(D^2+m)\delta_{12}. \end{aligned} \tag{4.1}$$

By doing the usual D-algebra transformations (cf. [5]) and replacing $f(k)$ by its asymptotic form $f(k)\approx \pi/\sqrt{k^2}$ we get

$$iS_{1a}(p) = \frac{16}{N}\int\frac{d^3k}{(2\pi)^3}\int d^2\theta\phi_a(-p,\theta)(D^2-m)\bar{\phi}_a(p,\theta)\frac{\sqrt{k^2}\sin^2(k\wedge p)}{((k+p)^2+m^2)(k^2+4m^2)}, \tag{4.2}$$

which, by power counting is only logarithmically divergent.

The graph shown in Fig. 1b contributes

$$\begin{aligned} iS_{1b}(p) &= \frac{4\pi}{N}\int\frac{d^3k}{(2\pi)^3}\int d^2\theta_1 d^2\theta_2\frac{\sin^2(k\wedge p)}{f(p-k)}\frac{1}{k^2+m^2}\Big[\frac{2m(p-k)^{\alpha\beta}}{(p-k)^2((p-k)^2+4m^2)} \\ &+ (\frac{1}{(p-k)^2+4m^2}-\frac{\xi}{(p-k)^2})C^{\alpha\beta}-\frac{1}{(p-k)^2}\Big(\frac{1}{(p-k)^2+4m^2} \\ &+ \frac{\xi}{(p-k)^2}\Big)(p-k)^{\alpha\beta}D^2+\frac{2mC^{\alpha\beta}}{(p-k)^2((p-k)^2+4m^2)}D^2\Big]\delta_{12} \\ &\times \Big[(D^2+m)D_{\beta 2}\delta_{12}D_\alpha\phi_a(p,\theta_1)\bar{\phi}_a(-p,\theta_2)-D_{\alpha 1}(D^2+m)\delta_{12}\phi_a(p,\theta_1)D_\beta\bar{\phi}_a(-p,\theta_2) \\ &+ (D^2+m)\delta_{12}D_\alpha\phi_a(p,\theta_1)D_{\beta_2}\bar{\phi}_a(-p,\theta_2)+D_{\alpha 1}(D^2+m)D_{\beta 2}\delta_{12}\phi_a(p,\theta_1)\bar{\phi}_a(-p,\theta_2)\Big]. \end{aligned} \tag{4.3}$$

Superficially S_{1b} contains linear divergences. However, the UV leading term of S_{1b}, after the D-algebra transformations, turns out to be proportional to

$$\int \frac{d^3k}{(2\pi)^3} \int d^2\theta \frac{k_{\beta\alpha} \sin^2(k \wedge p)}{(k^2)^{3/2}} C^{\alpha\beta} \phi_a(-p,\theta)\bar{\phi}_a(p,\theta), \tag{4.4}$$

which vanishes since $C^{\alpha\beta}k_{\beta\alpha} = 0$. Therefore iS_{1b} in Eq. (4.3) is only logarithmically divergent. To obtain this divergent part we delete the $4m^2$ terms in the denominators of (4.3) and replace $f(p-k)$ by its asymptotic form. We then have the sum of three contributions:

a) The term proportional to $2m$. After the commutation of $D_{\beta 2}$ with D^2 and the use of $D_{\beta 2}\delta_{12} = -D_{\beta 1}\delta_{12}$ it contributes with:

$$\begin{aligned} iS^{(1)}_{1b} &= \frac{8m}{N} \int \frac{d^3k}{(2\pi)^3} \int d^2\theta_1 d^2\theta_2 \sin^2(k \wedge p) \frac{1}{k^2+m^2} \\ &\times \Big[\frac{(p-k)^{\alpha\beta} + C^{\alpha\beta}D^2}{[(p-k)^2]^{3/2}}\Big]\delta_{12} \\ &\times D_{\alpha 1}D_{\beta 1}(D^2 - m)\delta_{12}\phi_a(p,\theta_1)\bar{\phi}_a(-p,\theta_2). \end{aligned} \tag{4.5}$$

We now apply the identity $D_{\alpha 1}D_{\beta 1} = k_{\alpha\beta} - C_{\alpha\beta}D^2$ which implies in

$$\begin{aligned} iS^{(1)}_{1b} &= \frac{8m}{N} \int \frac{d^3k}{(2\pi)^3} \int d^2\theta_1 d^2\theta_2 \sin^2(k \wedge p) \frac{1}{k^2+m^2} \\ &\times \Big[\frac{(p-k)^{\alpha\beta} + C^{\alpha\beta}D^2}{[(p-k)^2]^{3/2}}\Big]\delta_{12} \\ &\times [k_{\alpha\beta}D^2 + C_{\alpha\beta}k^2 - mk_{\alpha\beta} + mC_{\alpha\beta}D^2]\delta_{12}\phi_a(p,\theta_1)\bar{\phi}_a(-p,\theta_2). \end{aligned} \tag{4.6}$$

After contracting the loop into a point we arrive at the following divergent correction

$$\begin{aligned} iS^{(1)}_{1b} &= \frac{8m}{N} \int \frac{d^3k}{(2\pi)^3} \int d^2\theta_1 d^2\theta_2 \sin^2(k \wedge p) \frac{1}{k^2+m^2} \\ &\times \Big[\frac{-k^{\alpha\beta}k_{\alpha\beta} + C^{\alpha\beta}C_{\alpha\beta}k^2}{[(p-k)^2]^{3/2}}\Big]\delta_{12}\phi_a(p,\theta_1)\bar{\phi}_a(-p,\theta_2). \end{aligned} \tag{4.7}$$

Since $-k^{\alpha\beta}k_{\alpha\beta} + C^{\alpha\beta}C_{\alpha\beta}k^2 = 2k^2 - 2k^2 = 0$ the term proportional to $2m$ gives zero contribution.

b) The term proportional to $(\xi + 1)$ contributes with

$$iS^{(2)}_{1b}(p) = -\frac{8}{N}\phi_a(-p,\theta)(3D^2 - m)\bar{\phi}_a(p,\theta)(1+\xi) \int \frac{d^3k}{(2\pi)^3} \frac{\sin^2(k \wedge p)}{(k^2+m^2)((p-k)^2)^{1/2}}. \tag{4.8}$$

c) The term proportional to $(\xi - 1)$ contributes with

$$iS^{(3)}_{1b}(p) = -\frac{8}{N}\phi_a(-p,\theta)(D^2 + m)\bar{\phi}_a(p,\theta)(1-\xi) \int \frac{d^3k}{(2\pi)^3} \frac{\sin^2(k \wedge p)}{(k^2+m^2)((p-k)^2)^{1/2}}. \tag{4.9}$$

By adding the UV leading (logarithmically divergent) parts of $iS^{(2)}_{1b}$, $iS^{(3)}_{1b}$ the total divergent contribution to iS_{1b} is equal to

$$\begin{aligned} iS_{1b}(p) &= -\frac{16}{N} \int \frac{d^3p}{(2\pi)^3} \int d^2\theta [(2+\xi)\phi_a(-p,\theta)D^2\bar{\phi}_a(p,\theta) - m\xi\phi_a(-p,\theta)\bar{\phi}_a(p,\theta)] \\ &\times \int \frac{d^3k}{(2\pi)^3} \frac{\sin^2(k \wedge p)}{((k+p)^2 + m^2)\sqrt{k^2}}. \end{aligned} \tag{4.10}$$

The linearly divergent part of the graph given in Fig. 1c in any gauge, after the D-algebra transformations is

$$\propto \int \frac{d^3k}{(2\pi)^3} \int d^2\theta \frac{k_{\beta\alpha} \sin^2(k \wedge p)}{(-k^2)^{3/2}} C^{\alpha\beta} \phi_a(-p,\theta)\bar{\phi}_a(p,\theta), \tag{4.11}$$

which vanishes being proportional to $C^{\alpha\beta}k_{\beta\alpha} = 0$. However, there are logarithmically divergent contributions given by

$$\begin{aligned} iS_{1c}(p) &= \frac{32}{N} m \int \frac{d^3p}{(2\pi)^3} \int d^2\theta \phi_a(-p,\theta)\bar{\phi}_a(p,\theta) \\ &\times \int \frac{d^3k}{(2\pi)^3} \frac{\sin^2(k \wedge p)}{((k+p)^2 + m^2)\sqrt{k^2}}, \end{aligned} \tag{4.12}$$

coming from the graph in Fig. 1c.

We conclude that the contribution to the effective action arising from the sum of (4.2), (4.10) and (4.12) is also free of dangerous UV/IR mixing and has the form

$$\begin{aligned} iS_{\phi\bar{\phi}}(p) &= -\frac{16}{N}(1+\xi) \int \frac{d^3p}{(2\pi)^3} \int d^2\theta \phi_a(-p,\theta)(D^2 - m)\bar{\phi}_a(p,\theta) \\ &\times \int \frac{d^3k}{(2\pi)^3} \frac{\sin^2(k \wedge p)}{((k+p)^2 + m^2)\sqrt{k^2}} + fin, \end{aligned} \tag{4.13}$$

where fin denotes the remaining terms which are UV finite and possess at most a logarithmic UV/IR infrared divergence (actually, because of the sine factor no infrared divergence appears). We see that the quadratic UV/IR infrared divergence that occurred in the nonsupersymmetric version of the model, discussed in Section II, has disappeared under the present supersymmetrization. After integration of the planar part, $S_{\phi\bar{\phi}}$ becomes

$$S_{\phi\bar{\phi}}(p) = -\frac{4(1+\xi)}{N\pi^2\epsilon} \int \frac{d^3p}{(2\pi)^3} \int d^2\theta \phi_a(-p,\theta)(D^2 - m)\bar{\phi}_a(p,\theta) + fin. \tag{4.14}$$

This divergence can be canceled with the help of an appropriate counterterm which implies in the following wave function renormalization constant for kinetic term $\phi_a(D^2 - m)\bar{\phi}_a$:

$$Z = 1 + \frac{4(1+\xi)}{\pi^2 N\epsilon}, \tag{4.15}$$

so that in the gauge $\xi = -1$ the correction is finite.

V. THE GENERAL STRUCTURE OF DIVERGENCES AND THE ABSENCE OF DANGEROUS UV/IR MIXING

We have explicitly verified, that the two point vertex functions of the ϕ field up to first order in $\frac{1}{N}$ do not produce nonintegrable divergences. To further clarify the issue of renormalizability

up to order $1/N$, we start by calculating the superficial degree of divergence ω of an arbitrary graph γ. To that end, let us denote the number of vertices $iA^\alpha * (D_\alpha\phi_a * \bar{\phi}_a - D_\alpha\bar{\phi}_a * \phi_a)$ by V_1, of $A^\alpha * A_\alpha * \phi_a * \bar{\phi}_a$ by V_2, of $\eta * \bar{\phi}_a * \phi_a$ by V_3, and of $f(p)c' * D^\alpha[A_\alpha, c]_*$ by V_c. Furthermore let P_ϕ, P_A, P_η, P_c be the number of propagators $< \phi_a\bar{\phi}_b >, < A^\alpha A^\beta >\sim \frac{D^2}{k^2}$ and $< \eta\eta >$, $< cc' >\sim \frac{D^2}{\sqrt{k^2}}$, respectively. Each loop contributes to ω with 2 (3 for the integral over d^3k, -1 because the contraction of a loop into a point decreases the number of D^2-factors which could be converted to momenta by 1). Each ϕ_a or A_α propagator contributes with -1 while each vertex V_1 brings $\frac{1}{2}$ since it contains one spinor derivative and each vertex V_c brings $-\frac{1}{2}$ due to the factor $f(p)$. Therefore, ω is

$$\o = 2L - P_\phi - P_A + \frac{1}{2}V_1 - \frac{1}{2}V_c. \tag{5.1}$$

where L designates the number of loops in γ. By using the well known topological identity $L + V - P = 1$, this becomes

$$\o = 2 + P_A + P_\phi + 2(P_\eta + P_c) - \frac{3}{2}V_1 - \frac{5}{2}V_c - 2(V_2 + V_3) - P_A - P_\phi. \tag{5.2}$$

The number of propagators may be expressed in terms of the number of the external lines E_ϕ, E_A, E_η, E_c and of the total number of fields N_ϕ, N_A, N_η, N_c used to construct γ as

$$P_\phi = \frac{1}{2}(N_\phi - E_\phi); \quad P_A = \frac{1}{2}(N_A - E_A); \quad P_\eta = \frac{1}{2}(V_3 - E_\eta); \quad P_c = \frac{1}{2}(N_c - E_c). \tag{5.3}$$

It is then easy to verify that

$$N_\phi = 2(V_1 + V_2 + V_3); \quad N_A = V_1 + 2V_2 + V_c; \quad N_\eta = V_3; \quad N_c = 2V_c. \tag{5.4}$$

By replacing Eqs. (5.4) and (5.3) into (5.2), and after taking into account that $\o$ decreases by $\frac{N_D}{2}$ when N_D supercovariant derivatives are moved to the external lines, one arrives at

$$\o = 2 - \frac{1}{2}(E_\phi + E_A) - E_\eta - E_c - \frac{1}{2}N_D. \tag{5.5}$$

We immediately see that $\o$ in the theory cannot be larger than one (it would be two only for vacuum supergraphs but these contributions vanish due to the properties of the integral over Grassmann coordinates [7]). We also note that E_ϕ must be even in order to have an (iso)scalar contribution. By the same reason, E_A must either be even or if not it must be accompanied by an odd number of spinor supercovariant derivatives.

The case $\o = 1$ corresponds to $E_\phi = 2$, or $E_A = 2$, or $E_\eta = 1$, or $E_A = N_D = 1$ (with the number of all other external lines in each case being zero). However, we have already proved that the graphs with $E_\phi = 2$ (depicted in Fig. 1) are at most logarithmically divergent, that the sum of the graphs with $E_A = 2$ (which are depicted in Fig. 3) is finite and contributes only to the effective propagator of the gauge field. Besides that, the graph with $E_\eta = 1$ is a tadpole graph which automatically vanishes in the commutator case, whereas in the anticommutator

case its effect is only to fix m as being the mass of the ϕ superfield (compare with [5]). As for the graph corresponding to $E_A = N_D = 1$, which is formally linearly divergent, its contribution is proportional to $\int d^5 z D^\alpha A_\alpha$ which is of course zero.

From this discussion, we see that, up to the leading order of the $\frac{1}{N}$ expansion, all divergences in the theory are only logarithmic. It means that the quantum corrections in the theory are, up to this order, free from nointegrable infrared UV/IR singularities. We hope that a similar situation takes place at higher orders in the $\frac{1}{N}$ expansion.

There are more possibilities if $\not{o} = 0$, namely $E_A = 4$, or $E_\phi = 4$, or $E_A = E_\phi = 2$, or $E_\eta = 1, E_\phi = 2$, or $E_\eta = 1, E_A = 2$, or $E_\eta = 2$, or $E_A = 1, E_\phi = 2, E_D = 1$, or $E_A = 3, E_D = 1$. The cases with either $E_\phi = 4$ or $E_A = 4$ or $E_A = 3$ or $E_\eta = 1$, $E_A = 2$ are particularly dangerous because they are potentially logarithmically divergent but there is no available counterterm to absorb these divergences. However, one can explicitly verify that in all these cases the integrands associated to the divergent parts are odd in the loop momentum and therefore vanish under symmetric integration. Thus up to leading order of the $\frac{1}{N}$ expansion only the cases $E_\eta = 1, E_\phi = 2$, or $E_\phi = 2, E_A = 2$, or $E_A = 1, E_\phi = 2, E_D = 1$ imply in divergences.

This means that we can construct effective interaction terms for an effective Lagrangian of the gauge field A^α which are finite and proportional to $\int d^5 z \frac{1}{\sqrt{\Box}}(DA)^2 A^2$ and $\int d^5 z (DA) A^2$, which are needed to complete the induced noncommutative Maxwell and Chern-Simons Lagrangians. The graph with two external η fields of order N is given by Fig. 2, and we already showed that it is finite. As for the subleading graphs with two external η fields they could only modify the effective propagator of η field in higher orders of the $\frac{1}{N}$ expansion.

In the commutator case, due to the invariance of the action (3.1) under charge conjugation, the contributions proportional to $\eta A^\alpha A_\alpha$ vanish in any finite order of the expansion. In particular, at the first order in $\frac{1}{N}$ this result can be seen directly as it turns out to be proportional to $\int d^2\theta d^3 p_1 d^3 p_2 A^\alpha(p_1,\theta) A_\alpha(p_2,\theta)\eta(-p_1 - p_2,\theta)\sin(p_1 \wedge p_2)$, which evidently vanishes.

To sum up, in the leading order of $1/N$ the only logarithmic divergences in the theory are those ones proportional to $\phi_a \bar{\phi}_a$, which give origin to the wave function renormalization of the ϕ field, those ones proportional to $\eta \phi_a \bar{\phi}_a$, which, by a method similar to that employed in [5], can be shown to have the same Moyal structure as the corresponding vertex in the classical action (both in the commutator and anticommutator cases), and those which are proportional to $\phi_a \bar{\phi}_a A^\alpha A_\alpha$ and to $A^\alpha (D_\alpha \phi_a)\bar{\phi}_a$, $A^\alpha \phi_a (D_\alpha \bar{\phi}_a)$.

It is easy to verify that the Moyal structure of the quantum corrections proportional to $A^\alpha (D_\alpha \phi_a)\bar{\phi}_a$, $A^\alpha \phi_a (D_\alpha \bar{\phi}_a)$ is preserved. For example, in the commutator case each supergraph in such quantum corrections contains an odd number of the triple vertices, and therefore they will furnish a product of an odd number of sine factors; thus, as in [5], we find that the planar contribution could have only the form $\sin(p_1 \wedge p_2)$ where p_1 and p_2 are two of the three incoming momenta. Such phase factor also reproduces the corresponding Moyal structure in the classical action. However, an analogous proof of the same fact for the quartic correction $\phi_a \bar{\phi}_a A^\alpha A_\alpha$ is much more complicated. This problem will be considered elsewhere.

VI. SUMMARY

In this paper we studied the minimal and supersymmetric inclusion of fermions in the pure noncommutative CP^{N-1} model. Although for both situations a great improvement in the long

distance behavior of the gauge two point vertex function was achieved, the case of minimal coupling still presented an infrared nonintegrable singularity in the self-energy of the basic scalar field. To evade this problem the supersymmetric extension was also considered and we proved that, at least to $1/N$ order, the supersymmetric model is free from a dangerous UV/IR mixing. This is a strong indication that this supersymmetric model has a consistent perturbative expansion. The theory exhibits very nontrivial properties as the generation of a dynamics for the spinor connection superfield, with both the Maxwell and the Chern-Simons terms being generated by the quantum contributions. The ghost superfields which are generated also possess nontrivial dynamics; however, they contribute to the quantum corrections only in $\frac{1}{N^2}$ and higher orders.

The analysis of the ultraviolet behavior unveiled some interesting aspects of the renormalization program for the two versions of the model. In both cases considered, the model turns out to be renormalizable since the use of a commutator or an anticommutator does not change the planar part of the amplitudes. All ultraviolet divergences are logarithmic and can be eliminated by adequate counterterms (the Moyal structure of the $\bar{\phi}_a\phi_a A^\alpha A_\alpha$ vertex still needs further analysis). Similarly to the noncommutative nonlinear sigma model, nontrivial wave function renormalizations for the auxiliary η and A_α fields are expected [5].

The wave function renormalization constant for the scalar superfield was shown to be gauge dependent whereas, due to charge conjugation invariance, the mixed two point vertex function of the A_α and η fields vanishes in the commutator case.

A further development of the model could consist in a more detailed investigation of the higher orders of the $\frac{1}{N}$ expansion. Also, it could be interesting to develop the extended supersymmetric generalization of this model by analogy with the $\mathcal{N} = 2$ super-Yang-Mills theory containing gauge and matter multiplets in $\mathcal{N} = 1$ description.

VII. ACKNOWLEDGMENTS

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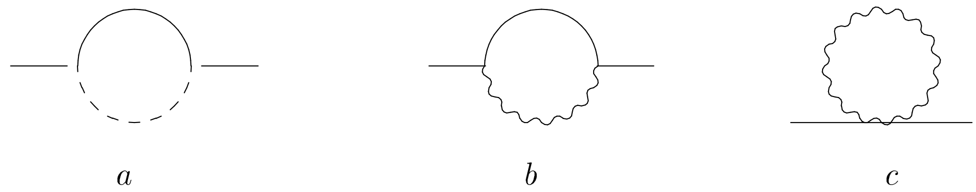

FIG. 1: Order $1/N$ contributions to the two point vertex function of the φ field. Continuous, dashed and wavy lines represent the propagators for the φ, λ and A_μ fields, respectively.

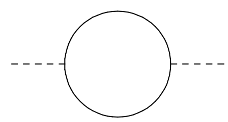

FIG. 2: Lowest order contribution to the propagator of the auxiliary η field. Here the dashed line is for η field and the continuous line for $\phi_a, \bar{\phi}_a$ fields.

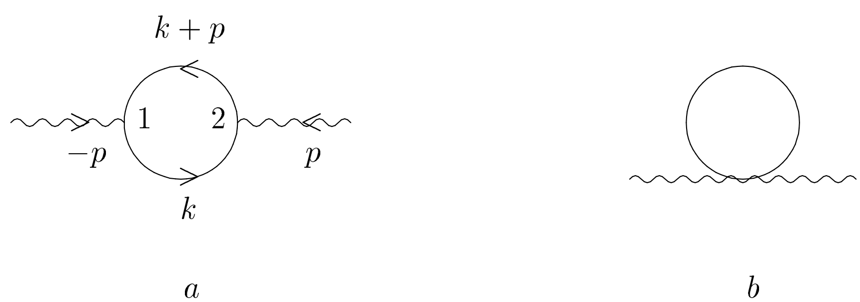

FIG. 3: Lowest order contributions to the propagator of the auxiliary A^α field.

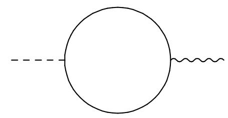

FIG. 4: A potential contribution to the two point vertex function of η and A_α fields.